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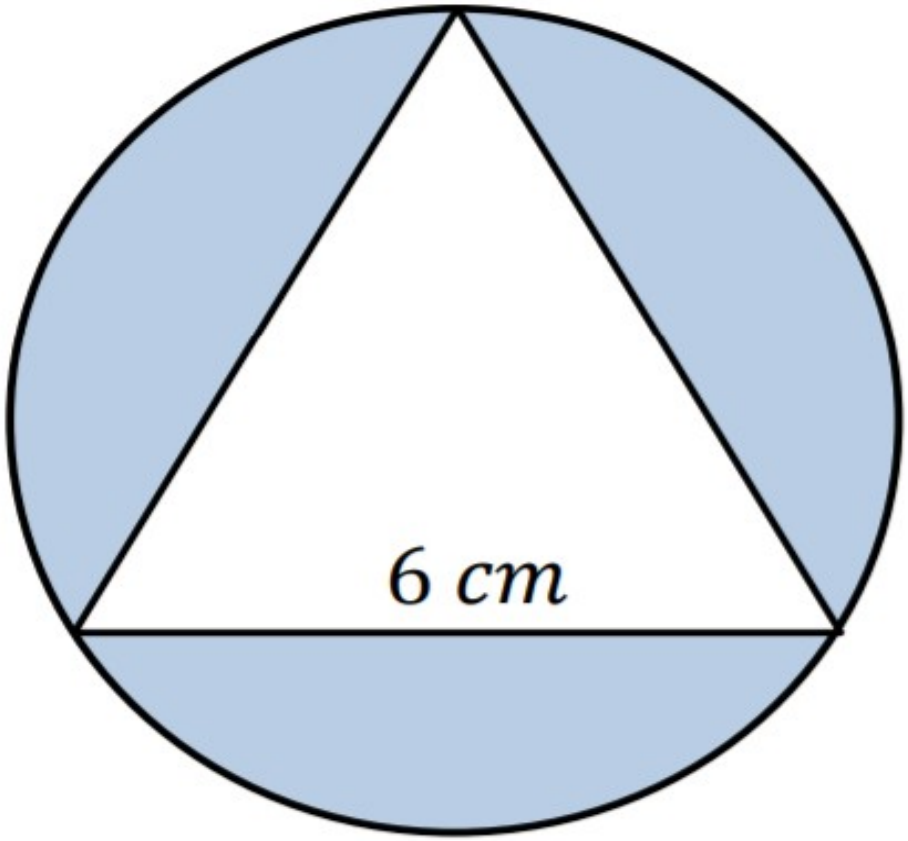
1.6 Circle Geometry – Starter Q

This diagram shows an equilateral triangle of side length 6 cm drawn inside a circle so that each corner touches the circumference of the circle.



What area of the circle is shaded?

C Coordinate geometry in the (x, y) plane

C2

Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$; completing the square to find the centre and radius of a circle; use the following properties:

- the angle in a semicircle is a right angle
- the perpendicular from the centre to a chord bisects the chord
- the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point.

C Coordinate geometry in the (x, y) plane

Assessed at AS and A-level

Teaching guidance

Students should be able to:

- find the equation of a tangent or normal at a point
- find relevant gradients using the coordinates of appropriate points.

Note: implicit differentiation of the equation of a circle will not be required at AS, but could be expected at A-level.

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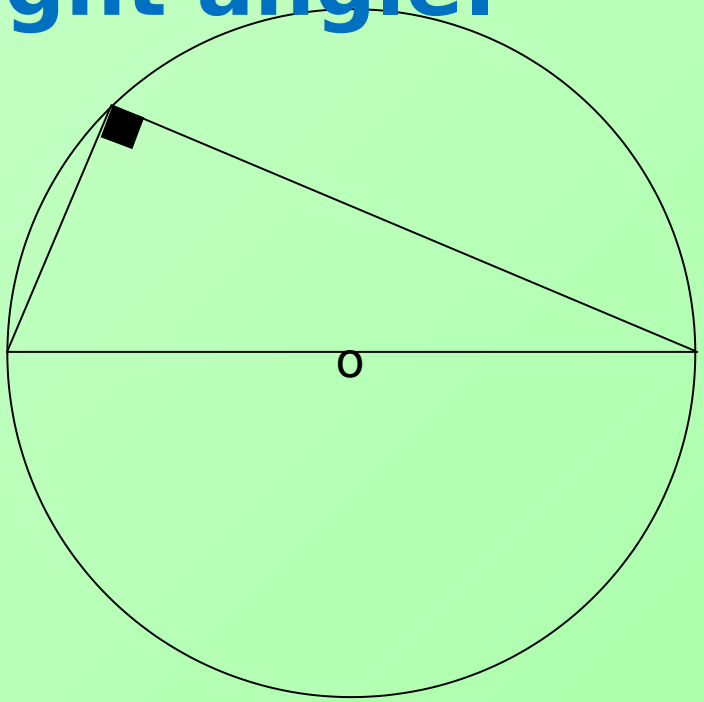
1.6 Circle Geometry

The angle in a semi-circle is a right angle.



1.6 Circle Geometry

Example 1

$A(-7,1), B(11,13)$ and $C(19,1)$ are three points on a circle. Prove that AC is a diameter.

If AC is a diameter, then by the angle in a semi-circle theorem, there must be a right-angle at B.

Gradient of AB:

Gradient of BC:

AB and BC are perpendicular hence AB is a diameter.

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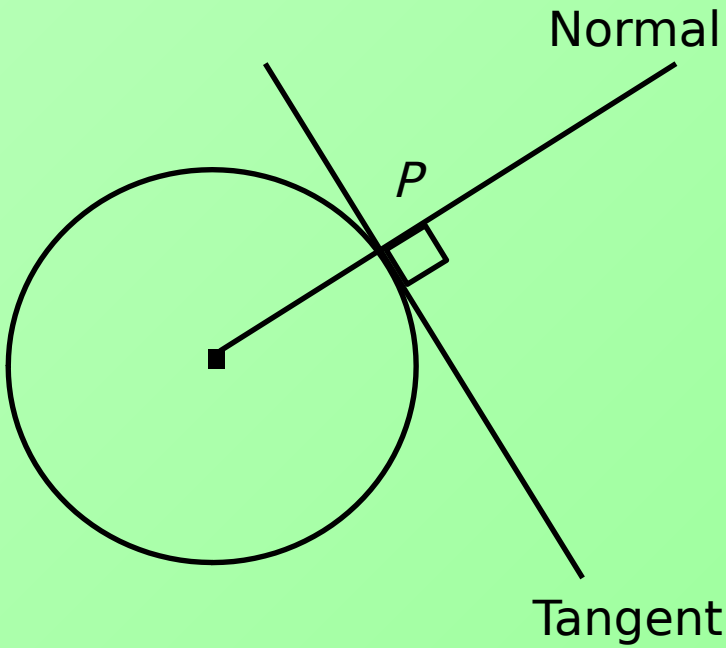
1.6 Circle Geometry

The **tangent** to a circle touches the
circle at just one point, P



The **normal** to a circle at any point P is **perpendicular** to the tangent at P and passes through the **centre** (i.e. same

1.6 Circle Geometry

Example 2ab

A circle has centre $C(2, -1)$ and radius 5. The point P has coordinates $(6, 2)$.

(a) Write down an equation of the circle. *(3 marks)*

(b) Verify that the point P lies on the circle. *(2 marks)*

$$(x-2)^2+(y+1)^2=25$$

Hence (6, 2) lies on the circle.

1.6 Circle Geometry

Example 2c

A circle has centre $C(2, -1)$ and radius 5. The point P has coordinates $(6, 2)$.

(c) Find the gradient of the line CP. *(2 marks)*

(d) (i) Find the gradient of a line which is perpendicular to CP. *(2 marks)*

(ii) Hence find an equation for the tangent to the circle at the point P. *(1 mark)*

Gradient of CP:

1.6 Circle Geometry

Example 2d

A circle has centre $C(2, -1)$ and radius 5. The point P has coordinates $(6, 2)$.

(c) Find the gradient of the line CP. *(2 marks)*

(d) (i) Find the gradient of a line which is perpendicular to CP. *(2 marks)*

(ii) Hence find an equation for the tangent to the circle at the point P. *(1 mark)*

erpendicular gradient

ence an equation for the tangent at P:

1.6 Circle Geometry

Example 3a

The circle C has equation $(x-2)^2+(y-6)^2=100$.

a Verify that the point $P(10, 0)$ lies on C.

b Find an equation of the tangent to C at the point $(10, 0)$, giving your answer in the form $ax+by+c=0$.

Hence P(10, 0) lies on the circle.

1.6 Circle Geometry

Example 3b

The circle C has equation $(x-2)^2+(y-6)^2=100$.

a Verify that the point $P(10, 0)$ lies on C.

b Find an equation of the tangent to C at the point $(10, 0)$, giving your answer in the form $ax+by+c=0$.

Centre:

Gradient of CP:

Perpendicular gradient:

Hence an equation for the tangent at P:

1.6 Circle Geometry

Example 4

A circle C has equation $(x - 5)^2 + (y + 3)^2 = 10$.

The line l is a tangent to the circle and has gradient -3.

Find two possible equations for l, giving your answers in the form $y = mx + c$.

We know:

- Centre is (5, -3)
- Radius is
- Gradient of the radius is

Let the point of intersection be

1.6 Circle Geometry

Example 4

A circle C has equation $(x - 5)^2 + (y + 3)^2 = 10$.

The line l is a tangent to the circle and has gradient -3.

Find two possible equations for l, giving your answers in the form $y = mx + c$.

We know:

- Centre is (5, -3)
- Radius is
- Gradient of the radius is

Let the point of intersection be

❶

1.6 Circle Geometry

Example 4

A circle C has equation $(x - 5)^2 + (y + 3)^2 = 10$.

The line l is a tangent to the circle and has gradient -3.

Find two possible equations for l, giving your answers in the form $y = mx + c$.

We know:

- Centre is (5, -3)
- Radius is
- Gradient of the radius is

Sub in ❶ to give:

Let the point of intersection be

1.6 Circle Geometry

Example 4

A circle C has equation $(x - 5)^2 + (y + 3)^2 = 10$.

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Find two possible equations for l, giving your answers in the form $y = mx + c$.

We know:

- Centre is (5, -3)
- Radius is
- Gradient of the radius is

Let the point of intersection be

Hence the POI could be

or so eqn for :

OR

1.6 Circle Geometry

Example 4

A circle C has equation $(x - 5)^2 + (y + 3)^2 = 10$.

The line l is a tangent to the circle and has gradient -3.

Find two possible equations for l, giving your answers in the form $y = mx + c$.

We know:

- Centre is (5, -3)
- Radius is
- Gradient of the radius is

OR

OR

Let the point of intersection be

OR

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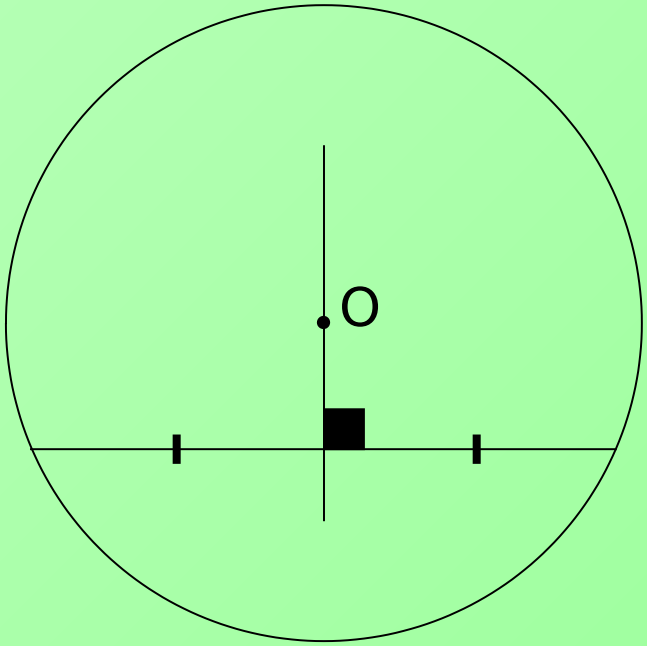
1.6 Circle Geometry

Chord bisector theorem:



The line perpendicular to the chord that passes through the centre, bisects the chord.

Or

The line bisecting the chord while passing through the centre, is perpendicular to the

1.6 Circle Geometry

Example 5a

The points $P(3, 16)$, $Q(11, 12)$ and $R(-7, 6)$ lie on the circumference of a circle. The equation of the perpendicular bisector of PQ is $y = 2x$.

a Find the equation of the perpendicular bisector of PR.

b Find the centre of the circle.

c Work out the equation of the circle.

perp. bisector of PR:

perpendicular gradient

Midpoint PR:

1.6 Circle Geometry

Example 5b

The points $P(3, 16)$, $Q(11, 12)$ and $R(-7, 6)$ lie on the circumference of a circle. The equation of the perpendicular bisector of PQ is $y = 2x$.

a Find the equation of the perpendicular bisector of PR.

b Find the centre of the circle.

c Work out the equation of the circle.

ntre of circle is the POI of the two bisectors:

Centre (3,6)

1.6 Circle Geometry

Example 5c

The points $P(3, 16)$, $Q(11, 12)$ and $R(-7, 6)$ lie on the circumference of a circle. The equation of the perpendicular bisector of PQ is $y = 2x$.

a Find the equation of the perpendicular bisector of PR.

b Find the centre of the circle.

c Work out the equation of the circle.

Equation of the circle is:

Given that P lies on the circle:

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1.6 Circle Geometry

Can you work out the shaded area in the diagram (the line shown just touches the smaller circle)?